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Spline interpolations besides Orskov model widely used in *in vitro* gas production

Mehmet Korkmaz

Department of Mathematics, Faculty of Sciences and Arts, Ordu University, Ordu, 52200, Turkey
Corresponding author e-mail: mkorkmaz52@yahoo.com

Abstract In this study, for the curve of *in vitro* gas production, three spline interpolations, which are alternative models passing through exactly all data points, with compare to widely used Orskov model applied to the data of *in vitro* gas production, were discussed. These models are linear spline, quadratic spline and cubic spline. The curves of *in vitro* gas production of spline interpolations and widely used Orskov model were shown on the same graph. Thus, these differences have been observed. The estimates for some intermediate values were done by using spline interpolations and Oskov model. Due to spline interpolations, the estimates of intermediate values could be made more precise. By using spline interpolations, the investigator is shown to obtain new ideas and interpretations in addition to the information of the well-known classical analysis

Keywords Cubic spline; gas production, linear spline, Orskov model, quadratic spline.

Introduction

It is known that the functions of regression models do not need to pass through all the data points; but they center all the data set. In other words, they reflect the general trend of change. In this study, some models passing through exactly all data points were mentioned. In this study, alternative models besides widely used Orskov model in *in vitro* gas production were investigated. An alternative model could be polynomial interpolation. But, piecewise polynomial interpolations will be presented in this study.

Although polynomial interpolation is valid on non-equally spaced discrete grids, it may develop a polynomial wiggle. There exists an alternative method to overcome the limitations of polynomial interpolation. If the entire interpolation interval is decomposed into smaller intervals connected at the given data points, the degree of interpolating polynomials can be reduced to avoid the polynomial wiggle. This idea leads to the spline interpolation [1]. For that reason, a polynomial between each consecutive two data points can be found. So, piecewise polynomials for all data set will be obtained. As known, if the lower degree polynomials are independent of each other, a piecewise approximation is obtained. An alternate approach is to fit a lower degree polynomial to connect each pair of data points and to require the set of lower degree polynomials to be consistent with each other in some sense. This type of polynomial is called a spline interpolation function or simply a spline [2].

Since the interpolation error can be made small even when using low degree polynomials for the spline, spline interpolation is preferred over polynomial interpolation.

Splines can be of any degree. Linear splines straight line segments connected each pair of data points. Linear splines are independent of each other from interval to interval. Linear splines yield first order approximating polynomials. The rates and curvature are discontinuous at every data point. Quadratic splines reveal second order approximating polynomials. The rates of the quadratic splines can be forced to be continuous at each data point, but the curvatures are still discontinuous. A cubic spline yields a third degree polynomial connecting each pair of data points. The rates and curvatures of the cubic splines can be forced to be continuous at each data point [2].

In order to find the additional conditions required to fit a cubic polynomial to two data points, these requirements are essential. One can define higher degree splines in a similar way. However, cubic splines have proven to be a good comprise between accuracy and complexity [2].

Cubic spline functions are the most popular spline functions, for a variety of reasons. They are a smooth function with which to fit data; and when used for interpolation, they do not have the oscillatory behavior that characterizes high degree polynomial interpolation. Splines are also fairly simple to calculate and use [3]. Consequently, cubic splines with linear and quadratic splines are given in this manuscript.

Instead of applying one polynomial to all data set, in order to minimize errors, this model is based on applying many polynomials (linear, quadratic or cubic) to the data in corresponding intervals [4].

By using these spline interpolation functions, the estimate could be made for the intermediate values between 3hours and 96 hours. Actually, cubic spline gives the best estimate value among the spline interpolation functions.

Some authors used cubic spline interpolations for lactation curves. But, firstly they determined several knot points instead of all lactation days and then they used cubic spline function [5]. But that cubic spline function did not pass the observed average daily milk yield for the lactation days**.** Nowadays, it is not difficult to construct spline functions which pass through all the data set, since it is easy to make calculations using the computers. Therefore, in this study the spline functions for passing through all the data set were investigated. Lagrange and Newton interpolation functions passing through all the data set were used in in vitro gas production [6]. But, in this study, spline interpolation functions were used for the curve of *in vitro* gas production.

In this study, for the curve of *in vitro* gas production, spline interpolations, alternative modeling passing to exactly all data points with respect to widely used Orskov model applied to the data of *in vitro* gas production will be presented. As known, for the n+1 data points (t_0,y_0), (t_0,y_0), (t_1,y_1), …, (t_n,y_n), a polynomial $P_n(t)$ of degree n or less can interpolate these points so that the curve of $P_n(t)$ passes through these n+1 points. If n is a large, there may be trouble: $P_n(t)$ may tend to oscillate for t between the nodes $t_0,t_1,\ldots,t_n$. Hence, there may be some numerical instabilities in this polynomial [7].

Material and methods

Material

Ozkan (2006) reported that in his study the nutritive values of hays from Trifolium prantense harvested at vegetative, flowering, seeding stages between the dates 15 April and 15 June 2005 were evaluated by chemical composition and *in vitro* gas production techniques. In addition, he specified that *in vitro* gas production were determined at 3,6,12,24,48,72 and 96h incubation times.

In this study, for the presentation of the spline interpolation functions, the mean values of *in vitro* gas production (ml/l g DM) of Trifolium prantense harvested at vegetative, flowering and seeding stages were used by Ozkan [8] and these mean values were presented in Table 1.

Methods

Although there are some researchers who have used the different mathematical models to *in vitro* gas production kinetics, the exponential model developed by Orskov and McDonald [9], $y = a+b(1-e^{-ct})$ is the most popular model in mathematical model used to describe the *in vitro* gas production kinetics where a is the gas production from the immediately soluble fraction (ml), b is the gas production from the insoluble fraction(ml), c is the gas production rate constant for the insoluble fraction (ml h^{-1}), tis the incubation time (h), y is the gas produced at time "t" and e is the base of natural logarithm [10, 11, 12, 13].

The conditions for linear, quadratic and cubic spline interpolation fit are that a set of linear, quadratic and cubic spline interpolation through the data points was passed by using new linear, quadratic and cubic spline interpolation in each interval, respectively [14].

Linear, quadratic and cubic spline interpolations are to be constructed to interpolate the equation (1). On each interval $[t_0,t_1],[t_1,t_2],\ldots,[t_{n-1},t_n]$, these spline interpolations are given by a different linear, quadratic and cubic polynomials, respectively.

$$S(t)=\begin{cases} S_0(t) & t\in[t_0,t_1] \\ S_1(t) & t\in[t_1,t_2] \\ \cdots\cdots\cdots\cdots & \\ \cdots\cdots\cdots\cdots & \\ \cdots\cdots\cdots\cdots & \\ S_{n-1}(t) & t\in[t_{n-1},t_n] \end{cases}$$
(1)

The polynomials S_{i-1} and S_i interpolate the same value at the point t_i and therefore the equation (2) is given

$$S_{i-1}(t_i)=y_i=S_i(t_i) \qquad (1\le i\le n-1)$$
(2)

Hence, S is automatically continuous [14].

Results

By using Table 1, linear spline, quadratic spline, cubic spline and the function of Orskov model are investigated, respectively.

The functions of linear, quadratic and cubic splines and Orskov model were tested for *in vitro* gas production (ml/l g DM) of Trifolium prantense harvested at vegetative, flowering, seeding stages. By using Table 1, linear, quadratic and cubic splines were given in Tables 2, 3 and 4, respectively and then the function of Orskov model was given in Table 5. It is known that the functions

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of linear, quadratic and cubic splines are piecewise polynomials. So the functions of linear, quadratic and cubic splines can be separately investigated for each interval. Furthermore, how to change these functions on each interval can be separately observed. But, the function of Orskov model does not give us separate results on each interval. This model gives only general result.

Table 1 Mean values of *in vitro* gas production (ml/l g DM) of Trifoliumprantense harvested at vegetative, flowering, seeding stages

	Harvest Time		
Hours	Vegetative	Flowering	Seeding
3	119.16	114.16	101.66
6	180.83	165.83	149.16
12	240.00	211.16	200.00
24	304.16	290.83	265.83
48	355.83	341.66	302.50
72	385.83	367.50	325.83
96	397.50	376.66	340.00

Table 2 Linear splines for vegetative, flowering and seeding

Splines	$S_{1v}(t)$ for vegetative	$S_{1f}(t)$ for flowering	$S_{1s}(t)$ for seeding
Linear	$\begin{cases} 57.49+20.57t, & 3\le t\le 6 \\ 121.66+9.86t, & 6\le t\le 12 \\ 175.84+5.35t, & 12\le t\le 24 \\ 252.49+2.15t, & 24\le t\le 48 \\ 295.83+1.25t, & 48\le t\le 72 \\ 350.82+0.49t, & 72\le t\le 96 \end{cases}$	$\begin{cases} 62.49+17.22t, & 3\le t\le 6 \\ 120.50+7.56t, & 6\le t\le 12 \\ 131.49+6.64t, & 12\le t\le 24 \\ 240.00+2.12t, & 24\le t\le 48 \\ 289.98+1.08t, & 48\le t\le 72 \\ 340.02+0.38t, & 72\le t\le 96 \end{cases}$	$\begin{cases} 54.16+15.83t, & 3\le t\le 6 \\ 98.32+8.47t, & 6\le t\le 12 \\ 134.17+5.49t, & 12\le t\le 24 \\ 229.16+1.53t, & 24\le t\le 48 \\ 255.84+0.97t, & 48\le t\le 72 \\ 283.32+0.59t, & 72\le t\le 96 \end{cases}$
where t is hour of *in vitro* gas production, $S_{1v}(t)$, $S_{1f}(t)$ and $S_{1s}(t)$ are *in vitro* gas production (ml/l g DM) of harvested Trifoliumprantense for linear spline.			

Table 3 Quadratic splines for vegetative, flowering and seeding

Splines	Quadratic
$S_{2v}(t)$ for vegetative	$\begin{cases} 119.16+10.04(t-3)^2, & 3\le t\le 4.5 \\ 48.75+22.01t-2.70(t-6)^2, & 4.5\le t\le 9 \\ 170.69+5.78t-0.01(t-12)^2, & 9\le t\le 18 \\ 201.68+4.27t-0.12(t-24)^2, & 18\le t\le 36 \\ 288.49+1.40t-0.003(t-48)^2, & 36\le t\le 60 \\ 319.33+0.92t-0.02(t-72)^2, & 60\le t\le 84 \\ 397.50-0.02(t-96)^2, & 84\le t\le 96 \end{cases}$

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$S_{2f}(t)$ for flowering	$\begin{cases} 114.16+8.46(t-3)^2, & 3\le t\le 4.5 \\ 57.17+18.11t-2.43(t-6)^2, & 4.5\le t\le 9 \\ 150.94+5.02t+0.25(t-12)^2, & 9\le t\le 18 \\ 156.20+5.61t-0.20(t-24)^2, & 18\le t\le 36 \\ 290.67+1.06t+0.01(t-48)^2, & 36\le t\le 60 \\ 310.25+0.80t-0.02(t-72)^2, & 60\le t\le 84 \\ 376.66-0.02(t-96)^2, & 84\le t\le 96 \end{cases}$
$S_{2s}(t)$ for seeding	$\begin{cases} 101.66+7.69(t-3)^2, & 3\le t\le 4.5 \\ 45.91+17.21t-1.95(t-6)^2, & 4.5\le t\le 9 \\ 131.85+5.68t+0.03(t-12)^2, & 9\le t\le 18 \\ 164.96+4.20t-0.15(t-24)^2, & 18\le t\le 36 \\ 263.37+0.82t+0.01(t-48)^2, & 36\le t\le 60 \\ 260.37+0.91t-0.01(t-72)^2, & 60\le t\le 84 \\ 340.00-0.03(t-96)^2, & 84\le t\le 96 \end{cases}$
where t is hour of *in vitro* gas production, $S_{2v}(t)$, $S_{2f}(t)$ and $S_{2s}(t)$ are *in vitro* gas production (ml/l g DM) of harvested Trifoliumprantense for quadratic spline.	

Table 4 Cubic splines for vegetative, flowering and seeding

Splines	Cubic
$S_{3v}(t)$ for vegetative	$\begin{cases} 52.18+22.33t-0.20(t-3)^3, & 3\le t\le 6 \\ 78.73+17.02t-1.80(t-6)^2+0.10(t-6)^3, & 6\le t\le 12 \\ 165.95+6.17t-0.04(t-12)^2-0.003(t-12)^3, & 12\le t\le 24 \\ 204.52+4.15t-0.13(t-24)^2+0.002(t-24)^3, & 24\le t\le 48 \\ 294.10+1.29t+0.01(t-48)^2-0.001(t-48)^3, & 48\le t\le 72 \\ 320.14+0.91t-0.03(t-72)^2+0.0004(t-72)^3, & 72\le t\le 96 \end{cases}$
$S_{3f}(t)$ for flowering	$\begin{cases} 57.36+18.93t-0.19(t-3)^3, & 3\le t\le 6 \\ 82.99+13.81t-1.71(t-6)^2+0.11(t-6)^3, & 6\le t\le 12 \\ 147.51+5.30t+0.29(t-12)^2-0.02(t-12)^3, & 12\le t\le 24 \\ 151.40+5.81t-0.25(t-24)^2+0.004(t-24)^3, & 24\le t\le 48 \\ 306.92+0.72t+0.04(t-48)^2-0.001(t-48)^3, & 48\le t\le 72 \\ 304.16+0.88t-0.03(t-72)^2+0.0004(t-72)^3, & 72\le t\le 96 \end{cases}$

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$S_{3s}(t)$ for seeding	$\begin{cases} 50.46+17.07t-0.14(t-3)^3, & 3\le t\le 6 \\ 68.94+13.37t-1.23(t-6)^2+0.07(t-6)^3, & 6\le t\le 12 \\ 127.13+6.07t+0.02(t-12)^2-0.005(t-12)^3, & 12\le t\le 24 \\ 166.88+4.12t-0.18(t-24)^2+0.003(t-24)^3, & 24\le t\le 48 \\ 272.89+0.62t+0.03(t-48)^2-0.001(t-48)^3, & 48\le t\le 72 \\ 260.31+0.91t-0.02(t-72)^2+0.0003(t-72)^3, & 72\le t\le 96 \end{cases}$
where t is hour of *in vitro* gas production, $S_{3v}(t)$, $S_{3f}(t)$ and $S_{3s}(t)$ are *in vitro* gas production (ml/l g DM) of harvested Trifoliumprantense for cubic spline.	

Table 5 *in vitro* gas production (ml/l g DM) of Trifoliumprantense harvested at vegetative, flowering, seeding stages for Orskov model

Orskov Model	Harvest Time		
	Vegetative	Flowering	Seeding
y	$390.33-306.34e^{-0.06t}$	$374.74-296.18e^{-0.05t}$	$331.68-265.60e^{-0.06t}$
a	83.99	78.56	66.08
b	306.34	296.18	265.60
c	0.06	0.05	0.06

By using Tables 2, 3 and 4, the rates of the functions of linear, quadratic and cubic splines could be found for each interval In Table 2, the rates of the linear splines for vegetative, flowering and seeding are the coefficients of the time, t, on each interval, respectively. Because of that, these rates are seen easily in Table 2. So the rates of linear splines were not given separately. For example, the rates of linear splines for vegetative, flowering and seeding were found as 20.57, 17.22 and 15.83 on the first interval. These rates have the largest values for the linear splines for vegetative, flowering and seeding, respectively. While the rate of linear spline for vegetative has the largest value, the rate of linear spline for seeding has the minimum value on the first interval. Similarly, by using Tables 3 and 4, the rates of the quadratic and cubic splines for vegetative, flowering and seeding could be found by taking the first derivative of the piecewise functions on each interval. While the rates of linear splines are constant on each interval, the rates of the quadratic and cubic splines are not constant on each interval. The rates of quadratic and cubic splines were given in Tables 6 and 7. For example, the rates of quadratic splines for vegetative, flowering and seeding were found as 30.11, 25.39 and 23.06 on the time 4.5 hours. These rates have the largest values for the quadratic splines for vegetative, flowering and seeding, respectively. While the rate of quadratic spline for vegetative has the largest value, the rate of quadratic spline for seeding has the minimum value on the time 4.5 hours. Furthermore; the rates of cubic splines for vegetative, flowering and seeding were found as 22.33, 18.93 and 17.07 on the time 3 hours. These rates have the largest values for the cubic splines for vegetative, flowering and seeding, respectively. While the rate of cubic spline for vegetative has the largest value, the rate of cubic spline for seeding has the minimum value on the time 3 hours.

Table 6 The first derivatives of quadratic splines for vegetative, flowering and seeding

The first derivatives of Splines	Quadratic
$D_{2v}(t)$ for vegetative	$\begin{cases} 20.07t-60.21, & 3\le t\le 4.5 \\ 54.38-5.40t, & 4.5\le t\le 9 \\ 5.99-0.02t, & 9\le t\le 18 \\ 9.87-0.23t, & 18\le t\le 36 \\ 1.67-0.01t, & 36\le t\le 60 \\ 3.40-0.03t, & 60\le t\le 84 \\ -0.04t+4.09, & 84\le t\le 96 \end{cases}$
$D_{2f}(t)$ for flowering	$\begin{cases} 16.93t-50.78, & 3\le t\le 4.5 \\ 47.24-4.85t, & 4.5\le t\le 9 \\ -0.87+0.49t, & 9\le t\le 18 \\ 15.03-0.39t, & 18\le t\le 36 \\ 0.41+0.01t, & 36\le t\le 60 \\ 3.37-0.04t, & 60\le t\le 84 \\ -0.03t+2.93, & 84\le t\le 96 \end{cases}$
$D_{2s}(t)$ for seeding	$\begin{cases} 15.37t-46.12, & 3\le t\le 4.5 \\ 40.62-3.90t, & 4.5\le t\le 9 \\ 4.97+0.06t, & 9\le t\le 18 \\ 11.51-0.30t, & 18\le t\le 36 \\ -0.26+0.02t, & 36\le t\le 60 \\ 1.97-0.01t, & 60\le t\le 84 \\ -0.06t+5.82, & 84\le t\le 96 \end{cases}$
where t is hour of *in vitro* gas production, $D_{2v}(t)$, $D_{2f}(t)$ and $D_{2s}(t)$ are the first derivatives of *in vitro* gas production (ml/l g DM) of harvested Trifoliumprantense for quadratic spline.	

Table 7 The first derivatives of cubic splines for vegetative, flowering and seeding

The first derivatives of Splines	Cubic

$D_{3v}(t)$ for vegetative	$\begin{cases} 22.33-0.59(t-3)^2, & 3\le t\le 6 \\ 38.26-3.54t+0.29(t-6)^2, & 6\le t\le 12 \\ 7.08-0.08t-0.01(t-12)^2, & 12\le t\le 24 \\ 10.41-0.26t+0.01(t-24)^2, & 24\le t\le 48 \\ 0.22+0.02t-0.002(t-48)^2, & 48\le t\le 72 \\ 4.75-0.05t+0.001(t-72)^2, & 72\le t\le 96 \end{cases}$
$D_{3f}(t)$ for flowering	$\begin{cases} 18.93-0.57(t-3)^2, & 3\le t\le 6 \\ 34.31-3.42t+0.33(t-6)^2, & 6\le t\le 12 \\ -1.69+0.58t-0.05(t-12)^2, & 12\le t\le 24 \\ 17.79-0.50t+0.01(t-24)^2, & 24\le t\le 48 \\ -2.89+0.08t-0.003(t-48)^2, & 48\le t\le 72 \\ 5.36-0.06t+0.001(t-72)^2, & 72\le t\le 96 \end{cases}$
$D_{3s}(t)$ for seeding	$\begin{cases} 17.07-0.41(t-3)^2, & 3\le t\le 6 \\ 28.15-2.46t+0.21(t-6)^2, & 6\le t\le 12 \\ 5.69+0.03t-0.02(t-12)^2, & 12\le t\le 24 \\ 12.68-0.36t+0.01(t-24)^2, & 24\le t\le 48 \\ -2.47+0.06t-0.002(t-48)^2, & 48\le t\le 72 \\ 3.79-0.04t+0.001(t-72)^2, & 72\le t\le 96 \end{cases}$
where t is hour of *in vitro* gas production, $D_{3v}(t)$, $D_{3f}(t)$ and $D_{3s}(t)$ are the first derivatives of*in vitro* gas production (ml/l g DM) of harvested Trifoliumprantense for cubic spline.	

Orskov Model is given with its parameters in Table 5 where y is *in vitro* gas production (ml/l g DM) of Trifolium prantense harvested at vegetative, flowering, seeding stages and t is hour of *in vitro* gas production for Orskov model.

According to Orskov model, the sum of the parameters, a and b gives the asymptotic value when time goes to infinity. The asymptotic values of *in vitro* gas production (ml/l g DM) of Trifolium prantense harvested at vegetative, flowering, seeding stages were found as 390.33, 374.74 and 331.68, respectively. However, any asymptotic value could not find for spline interpolation functions since generally, spline interpolation functions are not valid outside the range of the data. The estimates of spline interpolation functions for each data are exactly same with the observed value. So, the error sum of squares (SSE) of linear spline, quadratic spline and cubic spline functions are always zero. However, estimates of other models except Lagrange and Newton interpolation functions for each data are not exactly same with the observed value. So, the error sum of squares of other models except Lagrange and Newton interpolation functions are not zero.

The rates of Orskov model were given in Tables 8. For example, the rates of Orskov for vegetative, flowering and seeding were found as 14.30, 12.96 and 12.76 on the time 3 hours. These rates have the largest values for Orskov model for vegetative, flowering and seeding, respectively. While the rate of Orskov model for vegetative has the largest value, the rate of Orskov model for seeding has the minimum value on the time 3 hours.

By using Table 1, the estimates of linear spline, quadratic spline, cubic spline and Orskov model for *in vitro* gas production of Trifolium prantense harvested at vegetative, flowering, seeding stages were given in Table 9.

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Table 8 The first derivatives of *in vitro* gas production (ml/l g DM) of Trifoliumprantense harvested at vegetative, flowering, seeding stages for Orskov model

Harvest Time	Vegetative	Flowering	Seeding
The first derivatives of Orskov model	$16.87e^{-0.06t}$	$15.10e^{-0.05t}$	$15.14e^{-0.06t}$

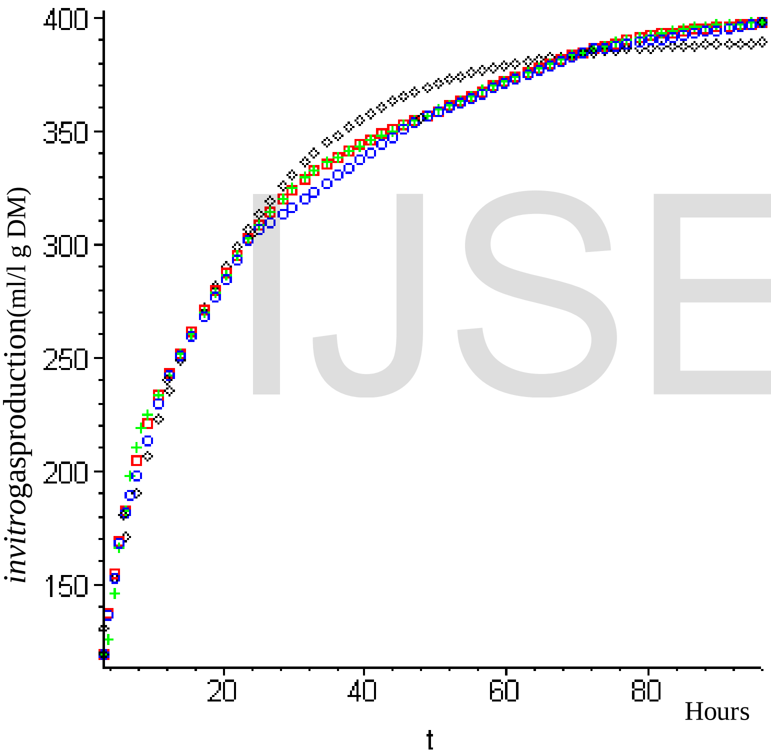
Table 9 The observed and estimated *in vitro* gas production (ml/l g DM) of Trifoliumprantense harvested at vegetative (V), flowering (F), seeding (S) stages according to linear spline, quadratic spline, cubic spline and Orskov model

Models	Harvest Time	Hour							
		3	6	12	24	48	72	96	SEE
Observed Mean *in vitro* gas production	V	119.16	180.83	240.00	304.16	355.83	385.83	397.50	
	F	114.16	165.83	211.16	290.83	341.66	367.50	376.66	
	S	101.66	149.16	200.00	265.83	302.50	325.83	340.00	
Linear Spline	V	119.16	180.83	240.00	304.16	355.83	385.83	397.50	0
	F	114.16	165.83	211.16	290.83	341.66	367.50	376.66	0
	S	101.66	149.16	200.00	265.83	302.50	325.83	340.00	0
Quadratic Spline	V	119.16	180.83	240.00	304.16	355.83	385.83	397.50	0
	F	114.16	165.83	211.16	290.83	341.66	367.50	376.66	0
	S	101.66	149.16	200.00	265.83	302.50	325.83	340.00	0
Cubic Spline.	V	119.16	180.83	240.00	304.16	355.83	385.83	397.50	0
	F	114.16	165.83	211.16	290.83	341.66	367.50	376.66	0
	S	101.66	149.16	200.00	265.83	302.50	325.83	340.00	0
Orskov	V	130.63	170.17	232.10	308.60	368.52	384.51	388.77	566.31
	F	120.58	156.63	214.12	287.63	349.12	367.20	372.52	217.74
	S	107.84	143.03	197.69	264.08	314.48	327.30	330.57	318.77

SSE: Error Sum of Squares

The estimates of the average *in vitro* gas production of Trifolium prantense harvested at vegetative, flowering, seeding stages of linear spline, quadratic spline and cubic spline were exactly same with the observed average *in vitro* gas production. So, the error sum of squares (SSE) of linear spline, quadratic spline and cubic spline functions were zero. But, the values of *in vitro* gas production of Trifolium prantense harvested at vegetative, flowering, seeding stages of Orskov model were more different. So, the error sum of squares of Orskov model for Trifolium prantense harvested at vegetative, flowering, seeding stages were found as 566.31, 217.74 and 318.77. These values were quite high.

The curves of *in vitro* gas production of spline interpolation functions and Orskov model for Trifolium prantense harvested at vegetative, flowering and seeding were given in the same graphs, Figures 1, 2 and 3, respectively. As shown in the Figures 1, 2 and 3, while all graphics were close to each other, the graphs of spline interpolation functions were very close to each other.



Color of linear spline: blue (o), color of quadratic spline: green (+), color of cubic spline: red (□), color of Orskov model: black (□)

Fig. 1.*in vitro* gas production for vegetative according to time (hour)

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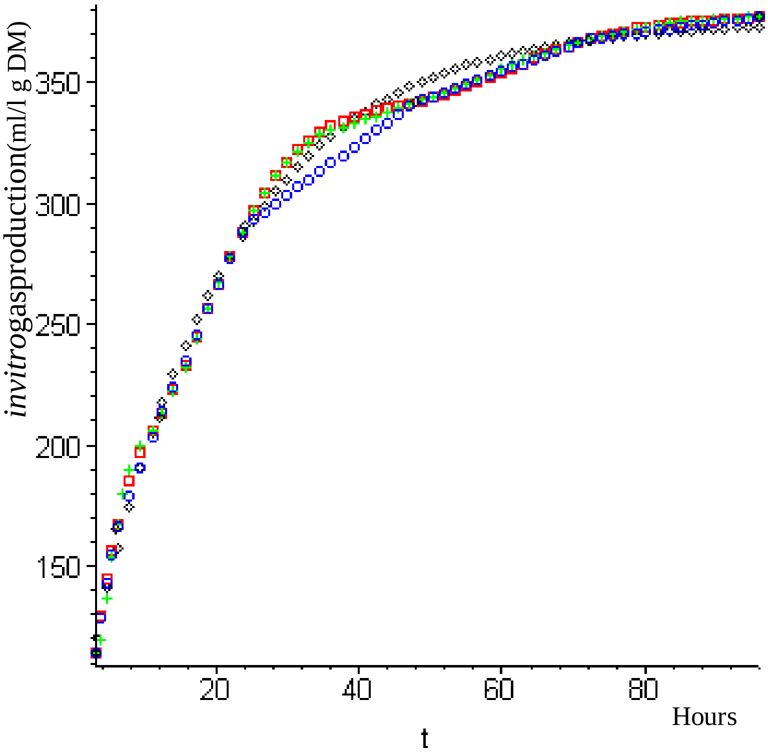
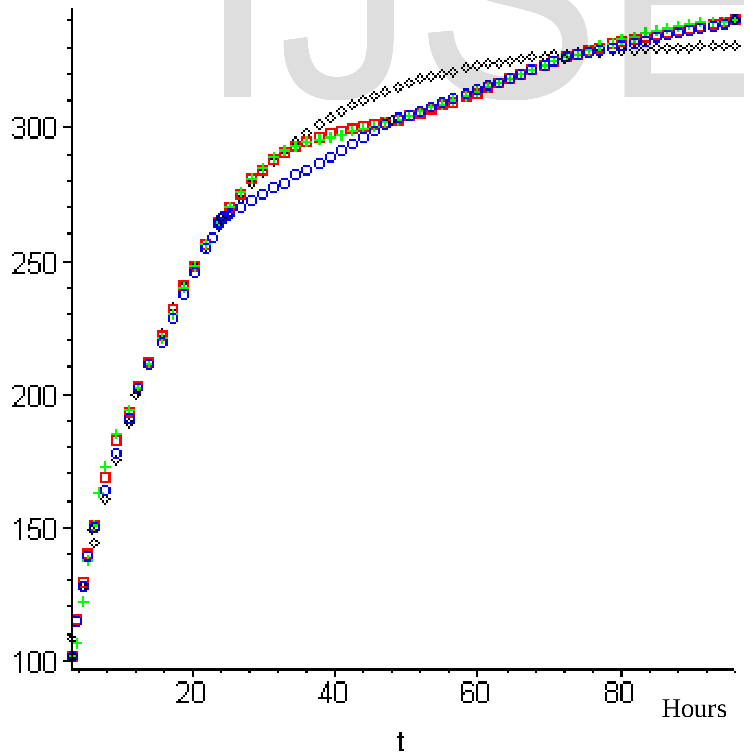


Color of linear spline: blue (o) , color of quadratic spline: green (+) , color of cubic spline: red (□) , color of Orskov model: black (□)

Fig. 2.*in vitro* gas production for flowering according to time (hour)



Color of linear spline: blue (o) , color of quadratic spline: green (+) , color of cubic spline: red (□) , color of Orskov model: black (□)

Fig. 3.*in vitro* gas production for seeding according to time (hour)

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The estimated values by using linear, quadratic and cubic splines are exactly same with the observed average *in vitro* gas production. But for Orskov model, the difference between observed and estimated average *in vitro* gas production is quite high.

Furthermore, by using the spline interpolation functions and Orskov model, the mean estimates for some intermediate hours of *in vitro* gas production of Trifolium prantense harvested at vegetative, flowering and seeding were found in Table 10.

Although the accuracy of these estimates could not be controlled, the estimates using especially cubic spline interpolation functions could be thought to be more accurate. For example, for the linear, quadratic, cubic spline interpolation functions and Orskov model, the estimated values of *in vitro* gas production of Trifolium prantense harvested at vegetative on the time 25 hours were found as 306.31, 308.31, 308.18 and 312.98, respectively. While the values found in the linear, quadratic and cubic spline interpolation functions are close to each other, the value found in Orskov model is far away from those three values.

Table10. The mean estimates for some intermediate hours of *in vitro* gas production of Trifoliumprantense harvested at vegetative (V), flowering (F), seeding (S) stages according to linear spline, quadratic spline, cubic spline and Orskov model

Models	Harvest Time	Hour				
		5	25	45	65	85
Linear Spline	V	160.27	306.31	349.37	377.08	392.15
	F	148.61	292.95	335.31	359.96	372.46
	S	133.33	267.36	297.92	319.03	333.51
Quadratic Spline	V	156.12	308.31	351.60	378.52	394.93
	F	145.29	296.24	338.53	361.06	374.82
	S	130.00	269.88	300.15	319.13	336.33
Cubic Spline.	V	162.24	308.18	352.02	378.32	394.00
	F	150.51	296.39	339.72	360.14	374.63
	S	134.70	269.78	30.86	318.73	334.89
Orskov	V	157.71	312.98	364.61	381.78	387.48
	F	145.22	291.96	344.89	363.97	370.85
	S	131.96	267.83	311.27	325.15	329.59

Discussion

Halang et al. [16] used cubic spline interpolation in their studies. In their studies, the cubic spline interpolations were shown on the graph in similar way to that in our study.

Sileshi et al. [17] and Dhanoa et al. [18] used linear spline (two-phase) model with a single unknown break point (knot) for fitting. Lopez et al. [19] used segmented model with three spline-lines delimited by two break points. Lopez et al. [19] reported that this model is a segmented model with two or three straight lines, and, thus, a sufficient number of observations are required in each segment to obtain a consistent solution. Nandraet al. [20] proposed an alternative in which modeling of substrate degradation was via the cubic smoothing spline, which is a smooth function (continuous in the first derivative) comprising piecewise cubic polynomials between sampling times. This statistical approach makes more difficult the interpretations of the gas production profiles and the estimation of the rate and extent of feed degradation in the rumen, limiting their applicability in routine measurements [21]. However, it could be say that different spline interpolation functions used for each consecutive two data points have the best fit according to other classical models.

Conclusion

Since in our study different spline interpolation functions were used for each consecutive two data points, the possible measurement error will not affect the entire data set. Since according to the study of literature, spline interpolation functions as in this study were not used, a better fit to all data set could be said in this study

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